

Bedform climbing in theory and nature

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ABSTRACT

Where bedforms migrate during deposition, they move upward (climb) with respect to the generalized sediment surface. Sediment deposited on each lee slope and not eroded during the passage of a following trough is left behind as a cross-stratified bed. Because sediment is thus transferred from bedforms to underlying strata, bedforms must decrease in cross-sectional area or in number, or both, unless sediment lost from bedforms during deposition is replaced with sediment transported from outside the depositional area. Where sediment is transported solely by downcurrent migration of two-dimensional bedforms, the mean thickness of cross-stratified beds is equal to the decrease in bedform cross-sectional area divided by the migration distance over which that size decrease occurs; where bedforms migrate more than one spacing while depositing cross-strata, bed thickness is only a fraction of bedform height.

Equations that describe this depositional process explain the downcurrent decrease in size of tidal sand waves in St Andrew Bay, Florida, and the downwind decrease in size of transverse aeolian dunes on the Oregon coast. Using the same concepts, dunes that deposited the Navajo, De Chelly, and Entrada Sandstones are calculated to have had mean heights between several tens and several hundreds of metres.

INTRODUCTION

Migrating bedforms and their deposits are dramatic features rich in geological information (Fig. 1). Since 1859, when H. C. Sorby realized that migrating bedforms deposit cross-strata like those preserved in rocks, geologists have attempted to relate the morphology of migrating bedforms to internal structures and external flow conditions. Inferring sediment transport directions from orientations of cross-strata is a classic technique in both modern and ancient environments, and identification of depositional environments is commonly based, in part, on interpretations of strata deposited by migrating bedforms.

To interpret cross-strata fully, however, requires *more than knowledge of the behaviour of individual bedforms*. Geologists must understand interactions between bedforms during deposition and erosion. For example, as will be discussed below, the thickness

of a cross-stratified bed deposited by a migrating bedform is controlled as much by differences in the sediment transport rate from one bedform to another as it is controlled by bedform size. Even bedform size depends on interactions between bedforms.

The analytical study of deposition by migrating bedforms was initiated by Allen (1963a, b, 1968, 1970), who suggested that cosets of cross-strata are deposited by trains of climbing bedforms. The purpose of this paper is to consider the theory of bedform climbing using a somewhat different approach from Allen's, to make a few simplifying assumptions that are most applicable to large two-dimensional bedforms, and finally to apply climbing theory to three examples in nature: tidal sand waves in St Andrew Bay, Florida, coastal dunes in Oregon, and dune deposits in Permian to Jurassic sandstones of the Colorado Plateau (Fig. 1). The main points of this paper are that, where certain flow conditions are



Fig. 1. Large-scale tabular sets of cross-strata in the Navajo Sandstone at Zion National Park, Utah. The sets are interpreted to have been produced by climbing aeolian draas or large dunes. No dune stoss surfaces are preserved between the bounding surfaces of the sets. Bus for scale.

met: (1) trains of large bedforms decrease in cross-sectional area as they deposit cosets of cross-strata; (2) large bedforms deposit cross-stratified beds having mean thicknesses that are generally only a small fraction of bedform height; and (3) sizes of two-dimensional bedforms that deposited cross-stratified beds can be estimated from bed thickness and downcurrent extent of deposition.

Bedforms can climb in unidirectional or oscillatory flows in either air or water, regardless of bedform size or shape. The following analysis of bedform climbing begins with a general treatment of climbing of all periodic bedforms but, for reasons discussed below, equations (10–17) and our quantitative conclusions are restricted to large, transverse, two-dimensional bedforms and their deposits. However, we use the deposits of small two-dimensional bedforms to illustrate qualitative aspects of bedform climbing, because deposits of small bedforms are more easily observed and photographed.

DEFINITIONS

As a bedform migrates, its trough moves through space, defining a surface. This surface separates the set of strata within the bedform from underlying strata and may therefore be called a bounding

surface (McKee & Weir, 1953). Where bedforms migrate downcurrent without net deposition, the volume of sediment deposited on lee slopes in a given time interval equals the volume of sediment eroded from stoss slopes. Hence, except for variations in trough elevation from bedform to bedform, the bounding surfaces generated by all troughs coincide. In a flow where net deposition is occurring, however, the volume of sediment deposited on each lee slope is greater than the volume eroded from the next stoss slope downcurrent. Consequently, bedforms and bounding surfaces move upward (climb) with respect to the generalized sediment surface, which is defined as a smooth surface passing approximately midway between troughs and crests (Figs 2, 3A). Where the sediment surface slopes downcurrent, as it commonly does where deposition occurs, bedforms can climb relative to the sediment surface while migrating downslope. Sediment deposited on the lee slope of one bedform and not eroded during the passage of a following bedform trough is left behind as a deposit called a pseudostratum (McKee, 1965) or a climbing translent stratum (Hunter, 1977a). The deposit is, in general, visibly cross-stratified and thus may also be called a cross-stratified bed or a set of cross-strata.

Much of the following discussion is restricted to two-dimensional transverse aeolian dunes and to the

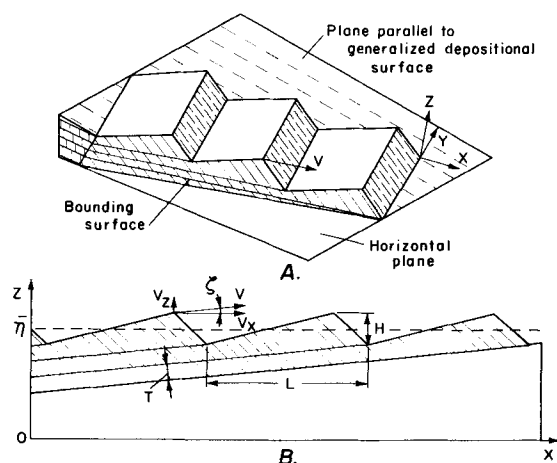


Fig. 2. Diagram illustrating climbing-bedform parameters. (A) Block diagram showing coordinate axes. (B) Cross-section parallel to xz -plane. L is bedform spacing; H is bedform height; T is thickness of climbing trans-lent stratum; V is direction of bedform migration, with components V_x and V_z ; ξ is angle of climb; η is height of generalized sediment surface.

class or classes of two-dimensional subaqueous bedforms known as sand waves (Southard, 1975; Dalrymple, Knight & Lambiasi, 1978), linear mega-ripples (Boothroyd & Hubbard, 1975; Hine, 1975), type I megaripples (Dalrymple *et al.*, 1978), flat dunes (Znamenskaya, 1963), flattened dunes (Pratt & Smith, 1972), bars (Costello, 1974), and two-dimensional dunes (Costello & Southard, 1981). For the purposes of the following discussion these bedforms are collectively called sand waves. Hydrodynamic relations between these and other bedforms are discussed by the authors above and by Rubin & McCulloch (1980).

ALTERNATIVES TO CLIMBING

To be complete, any model which explains the origin of a sequence of cross-stratified beds must account for both the strata and their bounding surfaces. In addition, the model must keep to the obvious but commonly overlooked constraint, that, where a deposit is produced, more sediment is brought in than is removed. The bedform climbing model meets this constraint by requiring only that the sediment transport rate decreases downcurrent or through time. Each erosional bounding surface is scoured by a

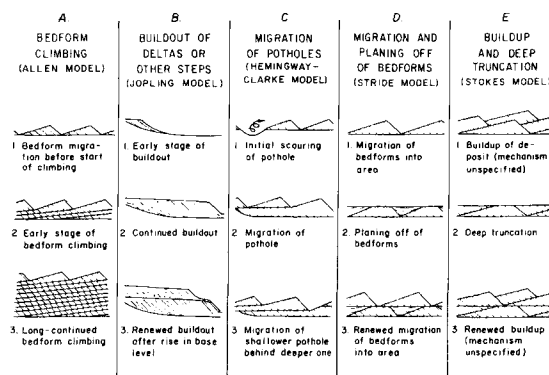


Fig. 3. Models for the formation of cosets of cross-strata (i.e. sets of cross-stratified beds or sets of climbing trans-lent strata). Models discussed in text.

migrating bedform trough, and each overlying set of cross-strata is deposited by a following bedform.

Perhaps the conceptually most simple alternative to climbing for the formation of a coset of cross-strata is the successive progradation of delta-like sand bodies across a depositional site (Fig. 3B). This type of deposit, which has been modelled in flume experiments by Jopling (1965) and has been documented from sedimentary sequences (Sykes & Brandy, 1976), requires sudden rises in the base level of deposition. Without such rises in base level, delta progradation produces only solitary cross-stratified beds.

Other alternatives to Allen's (1963a) model of bedform climbing have been proposed by Stride (1965), Hemingway & Clarke (1963), and Stokes (1968). Unlike bedform climbing, which can be a continuous process, the alternatives require alternating periods of deposition and truncation. For example, Stride suggested that sand waves deposit cross-strata during spring tides when current velocities are high. During neap tides ripples become the stable bed configuration, sand-wave crests are smoothed off, and ripple- or flat-bedded sands fill sand-wave troughs (Fig. 3D). According to Stride, the resulting deposit 'will be a three-dimensional complex of current-bedded lenses with occasional intercalations of flat or ripple-bedded sands.' Moreover, the flat- or ripple-bedded sands must form lenses that are as thick as the sets of cross-strata and that have a lateral spacing equal to the sand-wave spacing (Allen, 1968, fig. 5.36a). Stride's model can account for a deposit having these characteristics, but it cannot account for cosets of cross-strata that do not contain lenses of flat- or ripple-bedded sand.

Although the planing off of sand-wave crests and the filling of troughs is a necessary part of Stride's model, this process is not sufficient to produce a deposit thicker than about one-half the height of a sand wave. In addition, sediment must be introduced to the depositional area at a higher rate than it is removed. According to Stride, sediment is brought in during spring tides, when sand waves are active. During these periods, presumably the transport rate decreases downcurrent to allow deposition in the form of new sand waves overlying the previously formed deposits. These two conditions, sand-wave migration and deposition, are precisely the two conditions that require sand waves to climb. Therefore, when a deposit is produced under conditions postulated by Stride, that deposit can be expected to include translantent strata deposited by climbing sand waves, in addition to lenses of sand deposited in sand-wave troughs during neap tides.

Hemingway & Clarke (1963) accepted bedform climbing as a mechanism to deposit tabular cosets of cross-strata, but they objected to climbing as a mechanism to produce festoon cross-bedding. Instead, they favoured 'pot-holing' (Simons, Richardson & Albertson, 1961), a process whereby cross-strata are deposited in relatively deep troughs of three-dimensional bedforms (Fig. 3C). Without bedform climbing, pot-holing is incapable of producing cosets thicker than pot-hole depths. However, where climbing bedforms are three-dimensional, pot-holing can be expected to be an important control of the thickness of individual sets.

Another alternative to bedform climbing was proposed by Stokes (1968) to explain the nearly parallel bounding surfaces of sets of aeolian cross-strata. Stokes suggested that these bounding surfaces were formed by erosion down to buried water-table surfaces (Fig. 3E). This model has two serious problems. First, the migration of some dunes in Stokes' model (his fig. 1) was arbitrarily stopped, while other active dunes were arbitrarily drawn superimposed on them. In addition to requiring this very peculiar dune behaviour, the sets of cross-strata Stokes drew are unlike those seen in aeolian sandstones. Because Stokes' dunes do not climb, *each* set of cross-strata he drew includes one or more dune stoss slopes. In aeolian sandstones, however, preserved stoss slopes are extremely rare. This discrepancy between the model and the observed strata is a serious problem with the water-table model.

The second problem with the water-table model was discussed by Stokes. He realized that he had 'no

explanation for the imperfect but, nevertheless, roughly equal spacing' of bounding surfaces in aeolian sandstones. However, this regularity can be expected of climbing translantent strata. As discussed above and as illustrated below by equation (11), the thickness of a climbing translantent stratum is a function of dune height and the rate of downwind decrease in the transport rate. Consequently, the thicknesses of climbing translantent strata can be expected to be relatively constant when they are deposited by a train of two-dimensional regular dunes moving through an area in which the wind is relatively steady.

Stokes' model can be modified slightly to eliminate some of its weaknesses by assuming that the water table stands at or just below the level of the dune troughs while the dunes migrate without climbing. If the water table then quickly rises above the level of the dune troughs, interdune ponds appear, and sand that is stripped from the dune crests fills the interdune ponds. For a coset of cross-strata to form, new dunes must migrate in and the process be repeated. Although all but the last step in this sequence has been observed in modern dune trains (Hunter, 1977b), the modified Stokes model is geometrically identical to Stride's (1965) model (Fig. 3D) and suffers from the same geometrical defects when proposed as a general explanation for cosets of aeolian cross-strata.

RECOGNITION OF BEDFORM CLIMBING

When a bedform climbs at a steeper angle than the stoss slope of the bedform downstream (supercritical climb in the terminology of Hunter, 1977a, or what we refer to here by an alternative term, stoss-depositional climb), the entire bedform surface is preserved (Fig. 4A). Many examples of structures formed by stoss-depositional climbing ripples have been documented, not because such structures are especially common, but because preservation of the entire bedform makes climbing easy to recognize. In the vast majority of deposits, however, bedforms climb at subcritical or stoss-erosional angles and other criteria must be used to identify climbing. In the authors' experience less than 1% of climbing translantent strata deposited by wind ripples in dunes of the south Texas and Pacific coasts climb at stoss-depositional angles. From theoretical considerations

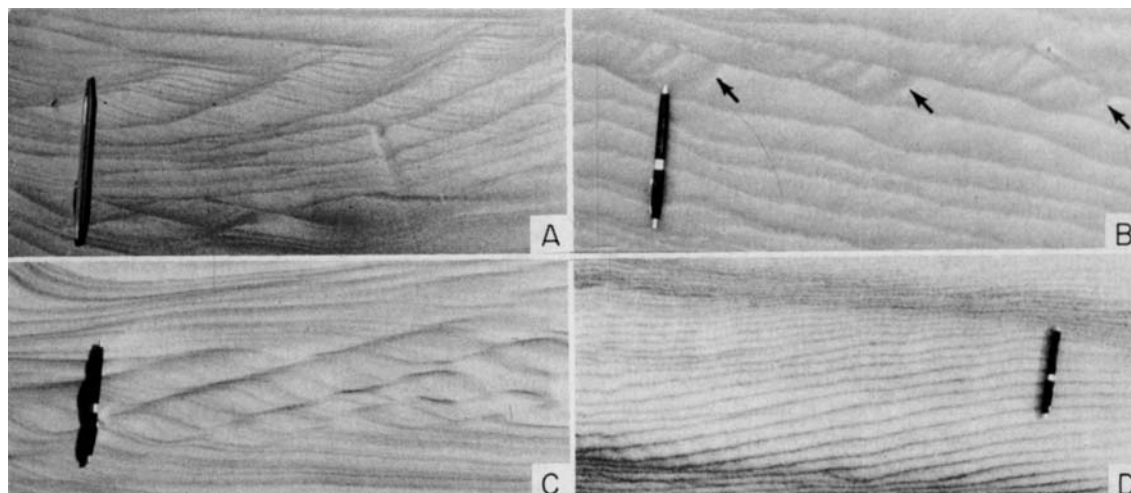


Fig. 4. Criteria by which climbing may be recognized. All photographs are of modern aeolian sands deposited by climbing wind ripples. In all photographs the exposure is horizontal, the sand was deposited on a surface that sloped toward the top of the photograph, and the ripples were migrating in an alongslope direction. Similar features can be seen in vertical exposures of deposits formed by climbing ripples. (A) Climbing recognizable by preservation of entire bedform surface in supercritically or stoss-depositional climbing ripples (near top of photograph). (B) Climbing recognizable by a few clearly correlative ripple-foreset cross-laminae (marked by arrows) in three adjacent translantent strata, which are otherwise without easily visible ripple-foreset cross-laminae. (C) Climbing recognizable by angular relation between the climbing translantent strata and the underlying and overlying isochronous strata. An exceptional example of a coset of cross-strata recognizable as having been deposited by climbing bedforms is present in a photograph by Nio (1976, fig. 7A). The figure illustrates a coset of cross-strata with bounding surfaces dipping upcurrent, similar to Fig. 4(C), but larger scale. (D) Climbing recognizable by change in the angle of climb of translantent strata, which have no visible ripple-foreset cross-laminae. Angle of climb is relatively low in the dark-coloured sand, relatively high in the light-coloured sand.

discussed below, stoss-depositional climb could be expected to be somewhat more common in subaqueous ripples but even less common in aeolian dunes and subaqueous sand waves.

Stoss-erosional climb is usually difficult to recognize, but under relatively rare circumstances structures formed by stoss-erosional climbing bedforms can be recognized by identifying distinctive layers that were deposited simultaneously on the lee slopes of several adjacent bedforms (Fig. 4B). To identify bedform climbing by this method requires that a distinctive type of sediment be delivered simultaneously to adjacent bedforms and requires an outcrop length of at least two, and preferably more, bedform spacings (tens of centimetres for ripples to as much as tens of kilometres for the largest aeolian dunes or draas).

Other features that can sometimes be used to recognize bedform climbing are: (1) an angular relation between climbing translantent strata and the isochronous or more nearly isochronous underlying or overlying strata or bounding surfaces of the set of translantent strata (Fig. 4C); and (2) a change in the

angle of bedform climb within a coset of translantent strata (Fig. 4D), the change having taken place simultaneously across the bedform field (Hunter, 1977b, fig. 5). Features of these types are quite common in structures formed by subaqueous and aeolian climbing ripples. They are much less recognizable in structures formed by large-scale subaqueous and aeolian bedforms, because, for reasons given below, such bedforms can usually be expected to climb at very low angles. Consequently, even a relatively large change in the angle of climb (a factor of two, for example) may not be visible.

Except for artificial marking of foresets by tracer grains, none of the climbing translantent strata that Hunter (1977b, fig. 3) observed in the process of formation contained any direct evidence of climbing. When direct evidence of climbing is not available, climbing can sometimes be inferred from indirect evidence. For example, climbing translantent strata deposited by trains of regular two-dimensional bedforms should be characterized by uniformity of thickness and large areal extent with respect to stratum thickness.

CLIMBING IN THEORY

Angle of climb

Climbing can be described by a vector of translation, V , which defines the speed and direction of bedform migration through three-dimensional space (Fig. 2A). Following Allen (1970), the vector V may be resolved into a component V_x parallel to the generalized depositional surface and equal to $V \cos \zeta$ and a component V_z normal to the surface and equal to $V \sin \zeta$ (Fig. 2B). The angle ζ is the angle of climb; the equation for its tangent is

$$\tan \zeta = \frac{V_z}{V_x}. \quad (1)$$

The component V_z is the net rate of deposition, defined as the rate of displacement of the generalized sediment surface in the z -direction. Thus,

$$V_z = \frac{\partial \eta}{\partial t} \quad (2)$$

where t is time, and η is the distance in the z -direction from the datum plane (where $z = 0$) to the generalized sediment surface. Spatial averaging over some convenient distance, which must be at least one bedform spacing, and averaging over some convenient time-scale are to be understood without special notation.

Equation (2) allows V_z to be expressed in terms of the sediment continuity equation (Middleton & Southard, 1978; Smith, 1977), which states the principle of conservation of sediment mass or, assuming constant bulk density, sediment bulk volume:

$$\frac{\partial \eta}{\partial t} = - \left(\frac{\partial i}{\partial x} + \frac{\partial c}{\partial t} \right) \quad (3)$$

where i is the spatially averaged bulk volume sediment transport rate across a unit length of depositional surface (dimensions of $l^3/l t = l^2/t$, where l is length), and c is the spatially averaged bulk volume of sediment in transport, either as bed load or suspended load, above a unit area of the depositional surface (dimensions of $l^3/l^2 = l$). Equation (3) states that at a point on the bed the rate of deposition ($\partial \eta / \partial t$) is equal to the rate of decrease of sediment transport with distance downstream ($-\partial i / \partial x$) plus the rate of decrease of the concentration of sediment in the flow through time ($-\partial c / \partial t$). Throughout the following discussion all sediment transport rates are bulk volume rates, and sediment porosity and density are assumed constant. The inclusion of only one

spatial partial derivative ($\partial i / \partial x$) in equation (3) implies the assumption that sediment transport is entirely in the direction of bedform migration; if this is not true, $\partial i / \partial x$ must be replaced by $(\partial i_x / \partial x + \partial i_y / \partial y)$, where i_x and i_y are the components of i in the x - and y -directions. It should be noted that Allen (1970, p. 18) gives an incorrect term in place of $\partial c / \partial t$.

The component V_x is the rate of bedform migration across the sediment surface. As shown in detail by Simons, Richardson & Nordin (1965), V_x can be related to the part of the sediment transport rate involved in bedform migration, hereafter called the *bedform transport rate*, i_b , and to the bedform height, H , by the equation

$$V_x = \frac{i_b}{kH} \quad (4)$$

where $k = A_c / LH$; A_c is the cross-sectional area of the bedform in the xz -plane, and L is the bedform spacing or wavelength; for triangular bedforms touching one another end-to-end, $k = \frac{1}{2}$. The bedform transport rate, i_b , plus what we will hereafter refer to as the *throughgoing transport rate*, i_t , make up the total transport rate, i (Fig. 5). As Middleton & Southard (1978) noted, the bedform transport rate is not necessarily equivalent to the bedload transport rate, because bedload as well as suspended load may bypass bedforms. For example, much of the bedload transport over aeolian ripples bypasses the ripples, does not contribute to ripple migration, and can therefore be called throughgoing transport. On the other hand, sediment suspended above the stoss slopes of large bedforms may be deposited on lee slopes and thereby contribute to bedform migration. Thus the bedform transport rate can either be greater than the bedload transport rate (where suspended sediment is deposited on lee slopes) or less than the bedload transport rate (where bedload sediment bypasses bedforms).

In equation (4) the migration rate is expressed in terms of the bedform transport rate averaged over one bedform spacing. In some studies the transport rate has been related to i_{bc} , the bedform migration rate at the bedform crest (Bagnold, 1941; Kachel & Sternberg, 1971; Wilson, 1972), or to the bedform transport rate across a fixed line that initially lies along the bedform crest (Bokuniewicz, Gordon & Kastens, 1977). The local bedform transport rate at the crest is related to the bedform migration rate by

$$V_x = \frac{i_{bc}}{H} \quad (5)$$

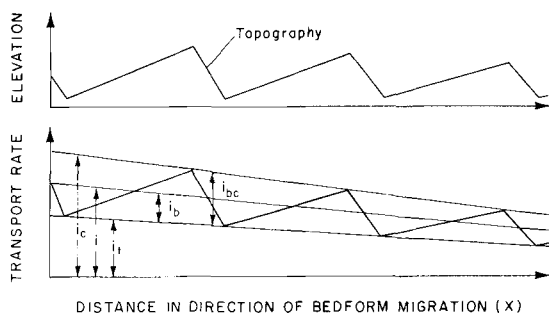


Fig. 5. Diagram illustrating bulk-volume sediment transport rates. The sawtooth transport rate curve represents the total transport over any point on the bed. i_t is transport across bedform troughs, or the throughgoing transport. i_c is total transport measured at a bedform crest. i_{bc} is the bedform transport rate measured at a bedform crest. i and i_b are respectively the total transport rate and the bedform transport rate averaged over one bedform spacing, L .

where i_{bc} is equal to $(i_c - i_t)$; i_c is the rate of sediment transport across the bedform crest, and i_{bc} represents that part involved in bedform migration (Fig. 5). The value of i_c is measurable only at bedform crests; elsewhere the value is interpolated.

As can be seen by comparing equations (4) and (5), and as Kachel & Sternberg (1971) implied, the average bedform transport rate of triangular bedforms is half the bedform transport rate at the crest. The various transport-rate and migration-rate equations have not always been adequately distinguished. For example, in relating aeolian dune migration to the sand transport predicted from wind measurements, Inman, Ewing & Corliss (1966) did not make it clear whether their wind measurements reflected flow over the dune crest or flow averaged over the entire dune.

Combining equations (1), (2), (3), and (4), it is found that

$$\tan \zeta = -\frac{kH}{i_b} \left(\frac{\partial i}{\partial x} + \frac{\partial c}{\partial t} \right). \quad (6)$$

An equally valid equation could be developed using equation (5) in place of equation (4). Equation (6) may usefully be modified to

$$\tan \zeta = -\frac{kH}{i_b} \cdot \frac{\partial i_b}{\partial x} (1 + A + B) \quad (7)$$

where $A = (\partial i_t / \partial x) / (\partial i_b / \partial x)$, and $B = (\partial c / \partial t) / (\partial i_b / \partial x)$. If transport is not entirely in the direction of bedform migration, A must be replaced by $A_x + A_y$, where $A_x = (\partial i_{tx} / \partial x) / (\partial i_b / \partial x)$, and $A_y = (\partial i_{ty} / \partial y) / (\partial i_b / \partial x)$. A large value of A_y helps to

produce large angles of climb in bedforms that are oblique or nearly longitudinal to the flow. The two dimensionless numbers A and B express the two ways in which high concentrations of suspended sediment can help produce high angles of climb. When the volume of sediment in transport above the bed at a given time (c) is large, the volume may decrease rapidly through time and thus produce large values of B and $\tan \zeta$. Similarly, when the rate of throughgoing sediment transport (i_t) is large, it may decrease rapidly downstream and thus produce large values of A and $\tan \zeta$.

The equation describing bedform climb simplifies considerably when A and B equal zero. The term A approaches zero in flows where bedforms are good traps for the sediment crossing their crests, or in other words where the bedform transport rate is large with respect to the throughgoing transport rate ($i_b \gg i_t$). This condition is approximated in many flume flows over medium- and coarse-grained sand waves that are not transitional with upper-regime flat beds (Simons *et al.*, 1965). Large sand waves in deep flows may or may not be better traps than small ones in shallow flows, depending on whether or not the deeper troughs and larger lee eddies of large bedforms are able to compensate in sediment-trapping ability for the increased transport above the bed in deep flows. Transverse aeolian dunes are especially good traps for the sediment crossing their crests (Wilson, 1972) because most transport occurs near the bed. The rarity or absence of large stoss-depositional climbing bedforms in the geological record suggests that large bedforms are indeed very good traps. On the other hand, the rate of throughgoing transport can approach or exceed the bedform transport rate in flows over antidunes, flows over sand waves that are transitional with upper-regime flat beds, flows over subaqueous ripples and fine-grained sand waves, flows over aeolian ripples, and flows in which abundant suspended sediment is inherited from previous or upstream conditions (e.g. turbidity currents).

The term B approaches zero in flows where deposition is caused primarily by a downstream decrease in the transport rate, not by a decrease through time. When sediment transport decreases through time but not downcurrent, the volume of sediment available to produce a deposit is limited to the volume in transport above the depositional site. In all but deep, turbid flows above small bedforms, the volume of sediment in transport above a bedform is much less than the volume of the bedform itself.

Where sediment transport decreases downcurrent, in contrast, the volume of sediment available for deposition (all sediment upstream from a depositional site) is essentially unlimited. When A and B both equal zero, as is approximately true for many flows over large two-dimensional bedforms,

$$\tan \zeta = -\frac{kH}{i_b} \cdot \frac{di_b}{dx}. \quad (8)$$

Thickness of a climbing translent stratum

If the direction of the bedform-climb vector and the attitude of the generalized depositional surface remain constant during deposition, the thickness, T , of a climbing translent stratum (Fig. 2B) is related to the angle of climb and to bedform spacing by the equation (Brookfield, 1977)

$$T = L \sin \zeta. \quad (9)$$

For angles of climb less than 17° , $\sin \zeta$ may be approximated by $\tan \zeta$ with no more than 5% error. Such angles include any subcritical or stoss-erosional angle of climb for almost any conceivable bedform. Using this approximation to relate equations (8) and (9),

$$T \approx -\frac{kHL}{i_b} \cdot \frac{di_b}{dx}. \quad (10)$$

The equation for T can be developed somewhat further for flows in which the bedform transport rate decreases linearly with distance downstream and in which bedforms are not created or destroyed. An approximately linear decrease in transport can be expected in many flows, but only in exceptional flows are bedforms not created or destroyed. Flows meeting these conditions are discussed first, not because they are typical, but because the analysis is simpler. Analysis of the case where bedforms are created and destroyed follows.

The assumptions of bedform conservation and steady flow conditions lead to the conclusion that bedform period (L/V_x , the time for a bedform to migrate one spacing) is constant. (Constancy of wave period where waves are neither created nor destroyed is demonstrated by Eagleson & Dean, 1966, pp. 21–24, for water waves, but their analysis applies equally to bedforms.) Where bedform period is constant, L is proportional to V_x and, because kHV_x is equal to i_b (equation 4), it follows from equation (10) that KHL/i_b is constant, and T is proportional to di_b/dx . Thus, where the bedform transport rate decreases linearly downcurrent, di_b/dx is constant, and

consequently the thickness (T) of a climbing translent stratum is constant downstream.

Assuming a linear decrease in the bedform transport rate from an upstream value, i_{bu} , at location x_u , to a downstream value, i_{bd} , at location x_d (Fig. 5), defining the distance from x_u to x_d as D , assuming $k = \frac{1}{2}$, substituting kHV_x for i_b (equation 4), and remembering that the bedform period is equal to L/V_x , equation (10) becomes

$$T \approx \frac{(HL)_u - (HL)_d}{2D}. \quad (11)$$

Where either H or L decreases linearly with distance downstream, and where transport out of a depositional area, represented by $(HL)_d$, is very small with respect to transport in, $(HL)_u$, it can be shown (see Appendix) that, with less than 10% error,

$$T \approx \frac{\overline{HL}}{D} \quad (12)$$

where the overbar denotes spatial averaging from x_u to x_d . Equation (12) states that the thickness of a cross-stratified bed is equal to twice the mean bedform cross-sectional area (area = $\overline{HL}/2$) divided by the length of the depositional area.

The thickness of a climbing translent stratum can be calculated by an independent method which does not involve assumptions of constancy of bedform number or period. As bedforms climb, the generalized sediment surface moves upward, and sediment once contained within bedforms is left as climbing translent strata. In other words, deposits gain sediment while bedforms lose sediment. In steady flows where bedforms are free to decrease in both size and number (Fig. 6), the cross-sectional area of a coset of cross-strata deposited by a train of bedforms (A_{coset}) is equal to the cross-sectional area of sediment lost by bedforms in the train. The cross-

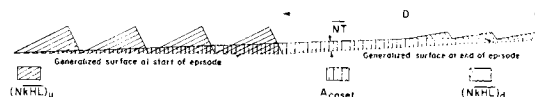


Fig. 6. Diagram illustrating deposition of a coset of cross-stratified beds by a train of bedforms free to decrease in both cross-sectional area, kHL , and number, N , while migrating a distance, D . $(NkHL)_u$ is cross-sectional area of bedforms at upcurrent end of area, $(NkHL)_d$ is cross-sectional area of same bedforms after their migration to downcurrent end of area, A_{coset} is cross-sectional area of the coset of cross-strata produced during migration of the bedforms, and T is thickness of a set. Diagram has extreme vertical exaggeration.

sectional area of sediment in the train at any time is equal to $Nk\overline{HL}$, where N is the number of bedforms in the train at that time, and $k\overline{HL}$ is their mean cross-sectional area. Thus,

$$A_{\text{coset}} = (Nk\overline{HL})_u - (Nk\overline{HL})_d, \quad (13)$$

In flows where each cross-stratified bed extends downcurrent for the entire distance that its depositing bedform migrated, the mean thickness of individual cross-stratified beds within a coset is equal to coset cross-sectional area divided by the distance, D , over which the observed decrease in $(Nk\overline{HL})$ occurs, and divided by \overline{N} , the mean number of bedforms in the train. Defining C as $(Nk\overline{HL})_d/(Nk\overline{HL})_u$,

$$\overline{T} \simeq \frac{(Nk\overline{HL})_u(1-C)}{\overline{N}D}. \quad (14)$$

In modern flows, N can be defined conveniently as the bedform frequency (V_x/L). Equation (14) is the general equation relating bed thickness to bedform size. Where the downstream cross-sectional area of the train is small compared to that upstream, C approaches zero, and equation (14) simplifies to

$$\overline{T} \simeq \frac{(Nk\overline{HL})_u}{\overline{N}D}. \quad (15)$$

Equations (14) and (15) illustrate that deposition can occur as the result of downcurrent decreases in size and/or number of bedforms migrating over the bed. Where the bedform number is constant, and where the bedform cross-sectional area linearly approaches zero downcurrent, the upstream bedform cross-sectional area is twice the bedform cross-sectional area averaged throughout the depositional area, and equation (15) reduces to

$$\overline{T} \simeq \frac{\overline{HL}}{D}. \quad (16)$$

Similarly, equation (14) reduces to equation (16) where bedform cross-sectional area is constant and bedform number linearly approaches zero downcurrent. Thus, whether or not bedform number is constant, where bedform transport into a depositional area is large compared to transport out, mean bed thickness is approximately equal to twice the mean bedform cross-sectional area divided by the length of the depositional area.

The two mechanisms discussed above decrease in bedform size and decrease in bedform number, should produce sand bodies with different characteristics. Where deposition occurs with a decrease in bedform size, the number of cross-stratified beds could be expected to remain relatively constant downcurrent. Where deposition occurs with a down-

current decrease in bedform number, the number of beds deposited during any time interval should decrease downcurrent. Where the transport rate decreases linearly downcurrent, the mean thickness of sets deposited by shrinking bedforms should be uniform throughout a depositional area; the mean thickness of sets deposited by bedforms decreasing in number should be greater downcurrent than upcurrent.

Perhaps the most useful application of bedform climbing theory is in calculating the heights of bedforms that deposited ancient cross-strata. Equation (16) can be modified for this purpose by substituting HI for L ; I is the bedform spacing-to-height ratio, called the bedform index. Solving for bedform height, equation (16) becomes

$$\overline{H} \simeq \left(\frac{\overline{T}D}{I} \right)^{\frac{1}{2}}. \quad (17)$$

Application of equation (17), using field observations to calculate heights of bedforms that deposited cross-stratified beds, is illustrated in the discussion of aeolian sandstones.

Bed thickness relative to bedform height

When interpreting cross-strata, some sedimentologists make what we feel to be an erroneous assumption, that bed thickness is nearly equal to bedform height. For example, Scheidegger & Potter (1967, p. 40) state 'It is assumed that the total height of a layer of cross-bedded sediments corresponds to the height of the sand-waves in a stream. Although not precisely true, the thickness of the vast majority of cross-bedded units appears to closely approximate the original dune height.' Scheidegger & Potter did not state how they determined heights of their dunes. Similarly, Brush (1965, p. 32) gave the more qualified suggestion that 'most, perhaps 80 %, of the dunes are represented in full height by their cross-stratified remanants. Obviously this statement is a subject worthy of discussion and perhaps future research.' Saunderson & Jopling (1980) made a similar but more qualified suggestion that set thickness is equal to four-fifths of bedform height.

Where sediment transport is entirely by downcurrent bedform migration, mean bed thickness can approach mean bedform height only where the depositional area is restricted in length to a single bedform spacing (equation 16). Equation (16) also explains why the largest modern aeolian dunes or draas are an order of magnitude greater in height than the thickest preserved aeolian cross-stratified

beds. Because depositional areas typically extend for many bedform spacings, bed thickness is likely to be only a small fraction of bedform height. The basic difficulty in preserving the total thickness of the lee-slope deposits is that total preservation requires stoss slopes to be non-erosional. Under this restriction, two problems arise: (1) providing an alternative source for sediment deposited on lee slopes, and (2) providing a mechanism for transporting that sediment without eroding stoss slopes.

Not all flows are limited by restrictions used in the above analysis, and bedforms may be largely preserved: (1) where sediment is deposited rapidly from suspension over a bedform train; (2) where bedforms in a train are three-dimensional or varied in size and climb at varying angles, possibly preserving some individual bedforms; or (3) where bedforms are longitudinal or oblique, rather than transverse. It would be desirable to have a method of estimating original bedform height independently of the method represented by equation (17). The only methods that we know of for estimating the original bedform height with reasonable accuracy are: (1) determining the bedform spacing from the angular relation between the climbing translational strata and isochronous surfaces, and dividing the spacing by an appropriate bedform index; and (2), where cross-strata are curved convex-up near the top of a set, extrapolating the curve to its maximum height by an appropriate mathematical formula.

Several characteristics of cross-strata seem to vary with bedform size or with vertical position on the bedform but are not yet well enough understood to serve as bases for estimating original bedform height. For example, the plan-view radius of curvature of cross-strata probably implies a certain maximum bedform size, because the contour lines on a bedform of given size probably cannot be curved more than a certain degree. As another example, the thickness and width of sandflow cross-strata probably vary with slipface height, at least in aeolian dunes, and the characteristics of sandflow cross-strata vary with vertical position on the slipface (Hunter, 1977b). As still another example, the proportions of sandflow, grainfall, slump, and other deposits probably vary with dune size and with vertical position on a dune (Hunter, 1977b).

Two controls of bedform size

In quasi-steady natural flows, bedform size is controlled by at least two kinds of processes –

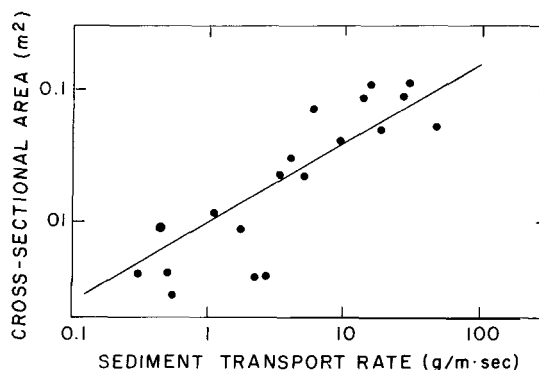


Fig. 7. Log-log plot of cross-sectional area of equilibrium sand waves as a function of measured sediment transport rate for all flows with 0.93 mm sand in data of Guy, Simons & Richardson (1966, table 6); bedform cross-sectional area is proportional to the transport rate raised to a power of 0.61. In flows with other sediment sizes, cross sectional area of equilibrium sand waves is proportional to the sediment transport rate raised to exponents commonly in the range of 0.25 to 3.0. Curve calculated by linear regression.

processes that determine equilibrium bedform size and processes that cause bedforms to change in size while acquiring or losing sediment during erosion or deposition. It is a remarkable coincidence, but apparently no more than coincidence, that when conditions change downcurrent in some natural flows, these two independent processes require a similar response.

When flow depth and sediment size are constant, the equilibrium size of bedforms in flumes increases with an increase in flow velocity, until the sand-wave and dune phase becomes transitional with the upper flat-bed phase (Yalin, 1964; Stein, 1965; Guy *et al.*, 1966, tables 2–5; Kennedy, 1969; Pratt & Smith, 1972). In a steady non-uniform flow of constant depth, therefore, equilibrium processes, like depositional-erosional processes, tend to make bedforms (that are not transitional with upper flat beds) grow smaller where a flow decelerates downcurrent and make them larger where flow accelerates.

Not only are the trends imposed by equilibrium processes and depositional-erosional climbing processes similar, but in some flume flows the two processes apparently can operate at similar rates (Fig. 7). In flows where bedforms are perfect traps, the climbing model requires that bedforms increase in cross-sectional area or in number in downcurrent-accelerating non-uniform flows. Where bedforms respond only by increasing in size downcurrent, the

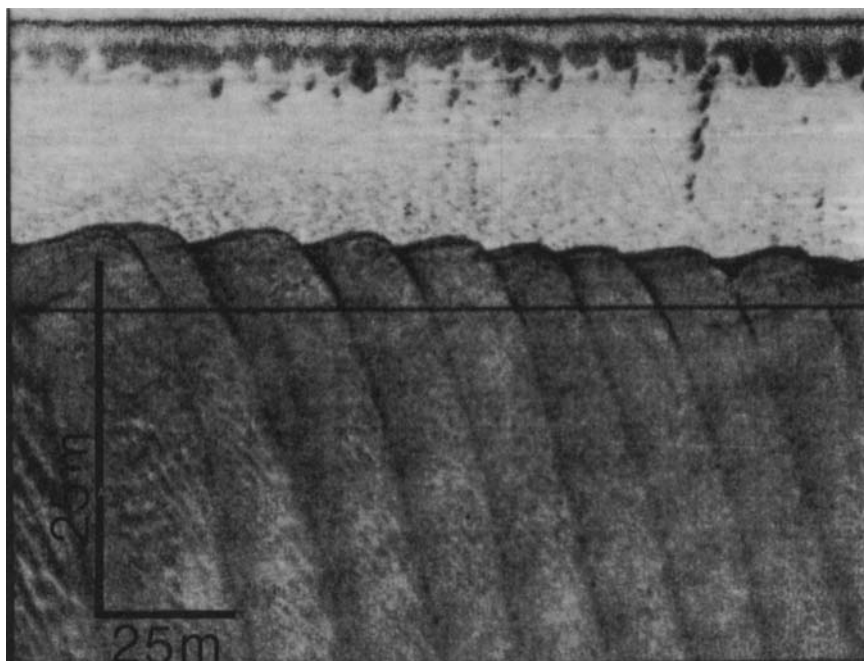


Fig. 8. Photograph of side-scan sonar record of two-dimensional bedforms decreasing in size downcurrent (left to right); from San Francisco Bay, California.

climbing model requires that average bedform cross-sectional area at different locations in a bedform train be proportional to the local bedform transport rate. Equilibrium processes observed in flumes indicate that, with some sediment sizes, the cross-sectional area of equilibrium bedforms is also approximately proportional to the sediment transport rate (Fig. 7). Thus, in some steady non-uniform flows, climbing could be expected to help keep bedform size in equilibrium or to considerably reduce the lag distance, which is the distance that bedforms must migrate before regaining equilibrium with a flow that has changed (Allen, 1974). Equilibrium processes and depositional-erosional processes require such a similar response that in some cases they may be indistinguishable. For example, superimposed bedforms observed at one instant in time commonly increase in height and spacing from the trough to the crest of larger bedforms on which they occur, in both subaqueous flows (McCave, 1971, fig. 5D) and aeolian flows (Fig. 8; Breed & Grow, 1979, figs 174A and D). This increase in size may be the result of bedforms growing to maintain equilibrium as flow accelerates from trough to crest of the large bedforms (Rubin & McCulloch, 1980). Alternatively, the superimposed bedforms may increase in size as

the result of negative climbing (bedform migration accompanied by net erosion).

The observation that aeolian dunes increase in size in some downstream-accelerating flows may be a clue that the equilibrium size of aeolian dunes, like the equilibrium size of subaqueous dunes in flumes, increases with flow velocity. Wilson (1972) suggested that the spacing of equilibrium dunes increases with wind speed, but to our knowledge there have been no empirical studies of the equilibrium size of aeolian dunes with which to test these ideas.

If the condition of constant flow depth, assumed in the previous discussion, is relaxed, then equilibrium processes and climbing processes may require conflicting bedform response. For example, in the flume data collected by Guy *et al.* (1966) sand waves and dunes in flows 30 cm deep are 1.5 to 2 times as large in cross-sectional area as those in flows 15 cm deep, for any constant transport rate. Consequently, in flows where the sediment transport rate is constant but depth changes downcurrent, equilibrium considerations may require that bedform size changes, and depositional-erosional considerations may require that size remain constant. What happens in nature has yet to be documented. One possibility is that climbing constraints cause changes in size to

lag behind downcurrent flow changes, until bedform size is so different from equilibrium size for local flow conditions that small bedforms coalesce or large ones split to form more nearly equilibrium-sized bedforms. Equilibrium processes are inferred to have the greatest possibility of overcoming climbing processes in: (1) flows in which bedforms are poor traps; (2) flows in which bedforms are frequently created or destroyed; and (3) flows where flow conditions change very gradually downcurrent.

Trains and fields of bedforms that exhibit systematic downcurrent variations in height and spacing occur in tidal flows in San Francisco Bay, California (Fig. 8), Long Island Sound, New York (Bokuniewicz *et al.*, 1977), Bay of Fundy, Canada (Middleton, personal communication), Vineyard Sound, Massachusetts (Briggs, Rubin & Southard, 1981), North Sea (Stride, 1970; McCave, 1971), Willapa Bay, Washington (Phillips, 1979) and St Andrew Bay, Florida (Salsman, Tolbert & Villars, 1965). In at least two examples downcurrent decreases in bedform size occur concurrently with deposition (Salsman *et al.*, 1965; Briggs *et al.*, 1981). In one example the pattern is reversed, and bedforms decrease in size in an erosional area (Phillips, 1979). In the North Sea, Stride (1970) concluded that sand-wave merging, rather than shrinking, provided sediment for deposition.

Like bedform size, bedform cross-sectional shape (particularly the concavity or convexity of stoss slopes) may be controlled, in part, by deposition and erosion. For example, where a downcurrent change in the transport rate results in bedforms containing too much sediment to attain equilibrium height and spacing, bedforms might develop stoss slopes that are more convex upward, as an alternative to increasing in height or spacing. Similarly, the bedform index (spacing-to-height ratio) may be influenced by downcurrent changes in the transport rate and the resulting acquisition or loss of sediment by bedforms.

Discussion of assumptions

Many of the assumptions made in developing this model of bedform climbing cannot confidently be expected to apply to three-dimensional bedforms. For example, of the numerous assumptions made above, we suspect that the one likely to cause the largest error is the assumption that all bedforms in a depositional area deposit beds that are preserved. This assumption can be expected to be a good approximation for trains of regular two-dimensional

bedforms, but where bedforms in a train are variable in size, and particularly where the angle of climb is extremely low, not all bedforms will leave deposits that survive the passage of following bedforms. In such a case, mean bed thickness can still be expected to be a fraction of bedform height, but calculated heights will be too large. Using modal bed thickness, rather than mean bed thickness, in equation (17) has the effect of eliminating or considerably reducing this error in cosets of cross-stratified beds where the thicknesses of individual sets follow normal or log-normal distributions.

The assumption that bedforms are perfect traps (A_z is zero) might also be expected to be most closely approximated in trains of two-dimensional bedforms. With any constant sediment size and flow depth, two-dimensional bedforms generally exist at lower flow velocities than do three-dimensional bedforms (Allen, 1968; Southard, 1975; Dalrymple *et al.*, 1978; Rubin & McCulloch, 1980). In the higher-velocity flows over three-dimensional bedforms, sediment could be more likely to bypass lee slopes in suspension. Furthermore, three-dimensional bedforms have discontinuous troughs and downcurrent-streaming spurs and ridges that leak sediment from one bedform to another. Where bedforms are not perfect traps, calculated bedform heights will be too large.

Another assumption made in developing this model is that bedform crests are normal to net transport (A_y is zero). An error in this assumption will produce calculated bedform heights that are either too large or too small, depending on whether the component of transport parallel to bedform crests decreases downcurrent (A_y is positive) or increases downcurrent (A_y is negative).

A few of the equations developed above are based on the assumption that the transport rate out of a depositional area is very small with respect to the transport rate in (C is zero). A failure of this assumption will result in calculated bedform heights that are too small. Despite the potential errors in making these assumptions, calculated bedform heights appear to be considerably better approximations of actual heights of two-dimensional bedforms than are the thicknesses of their sets of cross-strata.

BEDFORM CLIMBING IN NATURE

Sand waves in St Andrew Bay, Florida

This train of sand waves (Salsman *et al.*, 1965) is ideal for testing ideas about bedform climbing, not

only because many of the equation-simplifying assumptions appear to be met but also because Salsman *et al.* observed for two years bedform height, spacing, and migration rate at a site where there is direct evidence of bedform climb—an increase in mean bed level. They noted:

‘As each sand ridge passes a given point, however, many of the grains situated in the trough are left behind because the current transport capacity in the troughs is not great enough to remove all the grains deposited there by the previous ridge. The mean bed level is thus elevated a small amount as each ridge passes . . .’

‘About 120 cm beneath the trough level . . . the sediment type changes abruptly from sand to clay. This clay is apparently the “mud” referred to as the surface sediment in pre-1934 nautical charts. Thus, some 120 cm of sand have been deposited at the study site in the 30 years since the Corps of Engineers excavated the man-made channel. During this 30 years some ten ridges have passed the study site . . . Thus, assuming that bed-load transport is the primary mode of sediment movement at the site, each ridge has apparently left behind a layer of sand averaging 12 cm thick.’

The St Andrew Bay sand waves are also ideal because the following assumptions can be made.

(1) The sand waves occur in a tidal flow having peak spring-tide velocities of only 40 cm s⁻¹ at 55 cm above the bed. At these low velocities suspended sand transport probably is not significant, and the sand waves are probably very good sediment traps.

(2) According to Salsman *et al.*, the sand waves are ‘remarkably uniform’ transverse sand ridges. This property allows an assumption of two-dimensionality, and the extreme regularity shown in their fathometer profile (Salsman *et al.*, 1965, fig. 2) implies, for reasons discussed above, that sand waves are neither created nor destroyed at the depositional site (N is constant).

(3) The St Andrew Bay sand-wave migration period, L/V_s , is so long (3–4 y) compared to tidal periods (hours and weeks) that the tidal flow can be thought of as a steady flow, and the tidal periodicity as high-frequency noise that can be ignored.

The thickness of cross-stratified beds deposited by St Andrew Bay sand waves can be calculated using equation (14) and either of two observations made by Salsman *et al.* (1965): (1) changes in size undergone by a single bedform during migration, or (2) changes in size exhibited spatially by bedforms in a train observed at a single time. In the first case,

Table 1. Changes in size undergone by a migrating sand wave in St Andrew Bay, Florida (all dimensions in centimetres)

Upstream height	$H_u = 58$
Upstream spacing	$L_u = 1820$
Downstream height	$H_d = 49$
Downstream spacing	$L_d = 1780$
Depositional extent	$D = 1144$
Calculated bed thickness	$T = 8$

Table 2. Changes in size exhibited spatially by train of sandwaves in St Andrew Bay, Florida (all dimensions in centimetres)

Upstream height	$H_u = 58$
Upstream spacing	$L_u = 1820$
Downstream height	$H_d = 0$
Depositional extent	$D = 10^4$
Calculated bed thickness	$T = 5$

substituting in equation (14) the observed changes in size undergone by a single bedform during migration, mean bed thickness is calculated to be 8 cm (Table 1). In other words, the observed bedform lost sediment at a rate such that it deposited a cross-stratified bed 8 cm thick.

In the second calculation, using changes in size exhibited spatially by bedforms in a train, bed thickness is calculated to be 5 cm (Table 2). In this example equation (14) states that if all the sediment contained in the large upstream bedforms were distributed uniformly throughout the depositional area, layers 5 cm thick would be produced. Relative to the 58-cm sand-wave height, bed thicknesses calculated using equation (14) are in good agreement with the 12-cm thickness calculated by Salsman *et al.*, and all three calculated values are only a small fraction of bedform height. Differences between the three calculated values may reflect a non-linearity in the downcurrent decrease in the transport rate, unsteadiness caused by deposition of 120 cm of sediment during the last 30 y of bedform migration, differences in behaviour between individual bedforms, or the total disappearance of bedforms that reached the downcurrent end of the sand-wave field.

Modern aeolian dunes of the Oregon coast

Allen (1968, 1980) and Banks (1973) have illustrated what is probably a very common condition under which bedforms climb: the migration of bedforms down the lee slope of a larger bedform or isolated

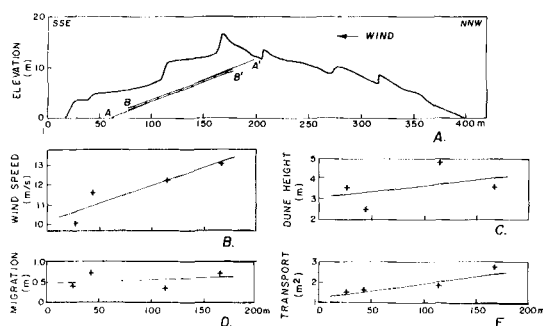


Fig. 9. Data documenting the climbing of dunes down the lee slope of a large sand mound near Lily Lake Road, Oregon. (A) Topographic profile and schematic portrayal of one set of cross-strata. Line A-A' represents the attitude of the generalized lee slope of the mound, and B-B' represents the calculated attitude and thickness of a set of crossbeds formed by the climbing of a dune down the lee slope from x_u to x_d . (B) Average velocities at 2 m above dune crests; each point represents average of three measurements at each of two points along a given dune crest. (C) Average dune heights (from toe of slipface to dune crest). (D) Average dune migration during the period of observation. (E) Average bulk-volume sediment transport at dune crests during the period of observation; in (C), (D), and (E), each point represents an average of four to nine measurements along a given dune. Curves in B-E calculated by linear regression analysis.

sedimentary mound. Brookfield (1977, 1979), in an application of this concept to an ancient aeolian sandstone, has interpreted sets of cross-strata bounded by so-called second-order surfaces as the products of dunes that climbed down the lee slopes of still larger bedforms (draas). A probable modern example of such a process can be seen in dunes of the Oregon coast just south of Lily Lake Road, about 13 km north of the town of Florence (latitude $44^{\circ} 05'N$, longitude $124^{\circ} 07'W$).

In the Lily Lake area, barchanoid dunes migrate to the south-south-east under the influence of nearly unidirectional sand-transporting winds during the dry summer months (Cooper, 1958, plate 3, locality 19). Each dune appears to be a nearly perfect trap for sand blown over the crest. The dunes migrate over a sand mound that is 16 m high and 380 m wide in the direction of sand transport (Fig. 9). Dune heights, dune migration rates, and wind velocities were measured at selected points, particularly on the lee slope of the mound, on 22 and 23 June 1979. The greatest wind speeds and greatest bedform transport rates (calculated by equation 4) were observed at the summit of the mound. Wind speed, dune height, dune migration, and bedform transport all showed a

general decrease down the lee slope of the mound (Fig. 9). If the wind conditions observed on the two days persisted for a sufficient length of time, the average angle of climb would have been 0.3° , and the average thickness of the climbing translent strata, calculated from equation (14), would have been 0.3 m, one-tenth the average dune height. A similar solution ($T = 0.4$ m) is obtained, without observations of dune migration, using the observed downwind decrease in dune height (Fig. 9c) and equation (14).

Aeolian sandstones

Existence of dune climbing

Aeolian sandstones are particularly attractive targets for applying the concepts of bedform climbing, because the tightly clustered, unimodal dip directions of many such sandstones have been interpreted as the result of deposition transverse dunes (McKee, 1979), which are relatively two-dimensional and nearly perfect sediment traps. The high trapping efficiency of aeolian transverse dunes follows from the fact that aeolian sand transport is dominantly in the form of saltation and creep and occurs very near the ground, especially when considered relative to the scale of the dune. Reasons for accepting a dominantly aeolian origin for various sandstones characterized by large-scale crossbedding, especially sandstones of Pennsylvanian to Jurassic age in the western interior United States, have recently been summarized by McKee (1979) and Hunter (1981).

The large-scale sets of crossbeds found in many aeolian sandstones are prime candidates for having been formed by the climbing of large dunes or draas, because they have several characteristics expected of the deposits of climbing bedforms: grouping of sets into a coset (Fig. 1), large length of a set (with respect to set thickness) in the direction of crossbed dip, and unimodality or acute bimodality of crossbed dip directions. Weber (1979), for example, found that the set length in the De Chelly Sandstone (Permian) averages 200 times the set thickness. Data summarized by McKee (1979, p. 195) show a narrow range of crossbed dip directions in many of the aeolian sandstones of the Colorado Plateau.

The tabular sets of crossbeds in aeolian sandstones have the additional expected characteristic of uniformity of set thickness within a coset, a feature noted by Stokes (1968) but not explained by his hypothesis. The trough-shaped sets of crossbeds have the additional expected characteristic of much greater

Table 3. Observations used to calculate heights of dunes or draas that deposited three aeolian sandstone formations

Formation	Bed thickness (T) (m)	Downcurrent depositional extent (D) (km)		Calculated dune heights (H) (m)
		Individual beds	Entire formation	
Navajo Sandstone	10*	1.6† /	300‡	33/450
De Chelly Sandstone	4.5§	0.9§ /	250‡	16/270
Entrada Sandstone	7¶	3.5¶ /	—	40/—

* Estimated from range of 6–15 m given by Peterson & Pipiringos (1979).

† Stokes (1968).

‡ From maps of Poole (1962).

§ Weber (1979).

¶ Kocurek (1981), first-order sets.

lateral extent in the direction of crossbed dip than in a transverse direction (Kocurek, 1981); this feature likewise is not explained by Stokes' hypothesis. Preserved stoss slopes between major bounding surfaces, a feature required by Stokes' hypothesis, occur very rarely if at all in the aeolian sandstones of the Colorado Plateau.

Definitive evidence that large-scale sets of crossbeds in aeolian sandstones were formed by bedform climbing is very difficult to establish, in part because the angles of climb were probably very low and in part because, as discussed below, the dunes or draas were probably very large. Recognition of bedform climbing becomes especially complicated where the bedforms are arranged in a hierarchy of sizes. Brookfield (1977, 1979), for example, identified three orders of sets of crossbeds in a Permian aeolian sandstone of Scotland, and he interpreted the first- and second-order sets as products of climbing draas and dunes, respectively. His interpretation of the second-order sets is based on a systematic angular relation between the second- and first-order bounding surfaces, but his interpretation of the first-order sets is, although reasonable, not based on definitive evidence.

The best documentation that we know of for the origin of first-order sets of aeolian crossbeds by the climbing of large dunes or draas is in Kocurek's (1981) study of the Entrada Sandstone (Jurassic) of north-east Utah and north-west Colorado. Kocurek shows that the first-order bounding surfaces climb (with respect to a formational boundary and to a thin shale bed that can be regarded as relatively isochronous) in the direction of crossbed dip. In addition, he presents evidence that interdune ponds and wet flats climbed with the large dunes or draas,

as hypothesized for modern dunes at White Sands by McKee & Moiola (1975). The modal angle of climb is only a few tenths of a degree.

Dune size

The size of dunes that deposited aeolian sandstones can be calculated by substituting in equation (17) a value of 15 for I , the bedform index (Wilson, 1972), and by using the measured bed thicknesses for T . In general, downcurrent depositional extent, D , cannot be determined accurately, because deposition may not have occurred simultaneously over the entire area covered by an aeolian sandstone. However, a lower limit for the depositional extent is the downcurrent extent of individual cross-stratified beds (Table 3). Using these values in equation (17), dunes that deposited the Navajo Sandstone (Triassic? and Jurassic), the De Chelly Sandstone (Permian), and the Entrada Sandstone (Jurassic) are calculated to have had mean heights greater than 16–40 m (Table 3). Actual heights may have been considerably higher.

If the total downcurrent extent of aeolian strata in a formation is used for depositional extent, calculated bedform heights are an order of magnitude higher: 270–450 m. These surprisingly high values suggest that: (1) deposition was localized and did not occur simultaneously over the entire area where each aeolian sandstone occurs; (2) these sandstones were deposited by oblique or longitudinal rather than transverse bedforms; or (3) the ancient bedforms were as large as the largest modern dunes or draas. This last possibility is not entirely unreasonable, because these sandstones contain the thickest aeolian cross-stratified beds that we know of (as much as 33 m in the Navajo Sandstone; McKee, 1979, p. 212).

Field observations made by Kocurek (1981) in the Entrada Sandstone are useful for comparison with the calculations made above. Kocurek determined mean bedform spacing to be 1500 m by measuring the distances between bounding surfaces along a stratigraphic horizon that he inferred was a depositional surface contemporaneous with the bedforms. Modern dunes having spacings of 1500 m have heights of about 100 m (Wilson, 1972), and it is reasonable to assume that the Entrada dunes also were approximately 100 m in height. Relative to the 7 m mean bed thickness, this value is in good agreement with the minimum height of 40 m calculated using equation (17). If bedform climbing theory were ignored and dune height were assumed to be equal to bed thickness, inferred bedform height would be too low by more than an order of magnitude.

CONCLUSIONS

- (1) Large transverse two-dimensional aeolian and subaqueous bedforms deposit cross-stratified beds by decreasing in size and/or in number while migrating downcurrent.
- (2) Bed thickness is usually a fraction of bedform height; the thickness of aeolian beds is only a small fraction of dune height.
- (3) The thickness of a cross-stratified bed can be calculated from changes in size exhibited by a single bedform during migration, and in special cases bed thickness can be calculated from variations exhibited spatially by bedforms in a train.
- (4) Where specified conditions are met, mean bedform height can be calculated from mean bed thickness, downcurrent bed extent, and bedform spacing/height index.
- (5) Dunes that deposited cross-stratified beds in the Navajo, De Chelly, and Entrada Sandstones are calculated to have had mean heights greater than several tens of metres and probably as much as several hundreds of metres.

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REFERENCES

- ALLEN, J.R.L. (1963a) Asymmetrical ripple marks and the origin of water-laid cosets of cross-strata. *Lpool Manchr geol. J.* **3**, 187-236.
- ALLEN, J.R.L. (1963b) The classification of cross-stratified units with notes on their origin. *Sedimentology*, **2**, 93-114.
- ALLEN, J.R.L. (1968) *Current Ripples: their Relations to Patterns of Water and Sediment Motion*. North-Holland, Amsterdam. 433 pp.
- ALLEN, J.R.L. (1970) A quantitative model of climbing ripples and their cross laminated deposits. *Sedimentology*, **14**, 5-26.
- ALLEN, J.R.L. (1974) Reaction, relaxation, and lag in natural sedimentary systems: general principles, examples, and lessons. *Earth Sci. Rev.* **10**, 263-342.
- ALLEN, J.R.L. (1980) Sand waves: a model of origin and internal structure. *Sedim. Geol.* **26**, 281-328.
- BAGNOLD, R.A. (1941) *The Physics of Blown Sand and Desert Dunes*. Methuen, London. 265 pp.
- BANKS, N.L. (1973) The origin and significance of some downcurrent dipping cross-stratified sets. *J. sedim. Petrol.* **43**, 423-427.
- BOKUNIEWICZ, H.J., GORDON, R.B. & KASTENS, K.A. (1977) Form and migration of sand waves in a large estuary, Long Island Sound. *Mar. Geol.* **24**, 185-199.
- BOOTHROYD, J.C. & HUBBARD, D.K. (1975) Genesis of bedforms in mesotidal estuaries. In: *Estuarine Research* (Ed. by L.E. Cronin), pp. 215-234. Academic Press, New York.
- BREED, C.S. & GROW, TERESA (1979) Morphology and distribution of dunes in sand seas observed by remote sensing. *Prof. Pap. U.S. geol. Surv.* **1052**, 253-302.
- BRIGGS, S.R., RUBIN, D.M. & SOUTHARD, J.B. (1981) Spatial variations in sand-wave size and migration rate: implications for shoal dynamics. *Bull. Am. Ass. Petrol. Geol.* **65**, 906.
- BROOKFIELD, M.E. (1977) The origin of bounding surfaces in ancient eolian sandstones. *Sedimentology*, **24**, 303-332.
- BROOKFIELD, M.E. (1979) Anatomy of a Lower Permian aeolian sandstone complex, southern Scotland. *Scot. J. Geol.* **15**, 81-96.
- BRUSH, L.M. JR (1965) Sediment sorting in alluvial channels. In: *Primary Sedimentary Structures and their Hydrodynamic Interpretation* (Ed. by G.V. Middleton). *Spec. Publ. Soc. econ. Paleont. Miner., Tulsa*, **12**, 53-56.
- COOPER, W.S. (1958) Coastal sand dunes of Oregon and Washington. *Mem. geol. Soc. Am.* **72**, 169 pp.
- COSTELLO, W.R. (1974) Development of bed configurations in coarse sands. *Mass. Inst. Technol. Dep. Earth planet. Sci. Rep.* **74**, 120 pp.
- COSTELLO, W.R. & SOUTHARD, J.B. (1981) Flume experiments on lower-flow-regime bed forms in coarse sand. *J. sedim. Petrol.* (in press).

- DALRYMPLE, R.W., KNIGHT, R.J. & LAMBIASE, J.J. (1978) Bedforms and their hydraulic stability relationships in a tidal environment, Bay of Fundy, Canada. *Nature*, **275**, 100–104.
- EAGLESON, P.S. & DEAN, R.G. (1966) Small amplitude wave theory. In: *Coastline Hydrodynamics* (Ed. by A.T. Ippen), pp. 1–92. McGraw-Hill, New York.
- GUY, H.P., SIMONS, D.B. & RICHARDSON, E.V. (1966) Summary of alluvial channel data from flume experiments from 1956 to 1961. *Prof. Pap. U.S. geol. Surv.* **462-I**, 11–196.
- HEMINGWAY, J.E. & CLARKE, A.M. (1963) Structure of ripple marks. *Nature*, **198**, 847–850.
- HINE, A.C. (1975) Bedform distribution and migration patterns on tidal deltas in the Chatham Harbor estuary, Cape Cod, Massachusetts. In: *Estuarine Research* (Ed. by L.E. Cronin), pp. 235–252. Academic Press, New York.
- HUNTER, R.E. (1977a) Terminology of cross-stratified sedimentary layers and climbing ripple structures. *J. sedim. Petrol.* **47**, 697–706.
- HUNTER, R.E. (1977b) Basic types of stratification in small eolian dunes. *Sedimentology*, **24**, 361–387.
- HUNTER, R.E. (1981) Stratification styles in eolian sandstones: some Pennsylvanian to Jurassic examples from the western interior U.S.A. In: *Recent and Ancient Non-Marine Depositional Environments: Models for Exploration* (Ed. by F.G. Ethridge and R.M. Flores), pp. 315–329. *Spec. Publs. Soc. econ. Paleont. Miner.*
- INMAN, D.L., EWING, G.C. & CORLISS, J.B. (1966) Coastal sand dunes of Guerrero Negro, Baja California, Mexico. *Bull. geol. Soc. Am.* **77**, 787–802.
- JOPLING, A.V. (1965) Laboratory study of the distribution of grain sizes in cross-bedded deposits. In: *Primary Sedimentary Structures and their Hydrodynamic Interpretation* (Ed. by G.V. Middleton). *Spec. Publ. Soc. econ. Paleont. Miner., Tulsa*, **12**, 53–65.
- KACHEL, N.B. & STERNBERG, R.W. (1971) Transport of bedload as ripples during an ebb current. *Mar. Geol.* **10**, 229–244.
- KENNEDY, J.F. (1969) The formation of sediment ripples, dunes, and antidunes. *Ann. Rev. Fluid Mechanics*, **1**, 147–169.
- KOCUREK, G. (1981) Significance of interdune deposits and bounding surfaces in aeolian dune sands. *Sedimentology*, **28**, 753–780.
- McCAVE, I.N. (1971) Sand waves in the North Sea off the coast of Holland. *Mar. Geol.* **10**, 199–225.
- McKEE, E.D. (1965) Experiments on ripple lamination. In: *Primary Sedimentary Structures and their Hydrodynamic Interpretation* (Ed. by G.V. Middleton). *Spec. Publ. Soc. econ. Paleont. Miner., Tulsa*, **12**, 66–83.
- McKEE, E.D. (1979) Ancient sandstones considered to be eolian. *Prof. Pap. U.S. geol. Surv.* **1052**, 187–238.
- McKEE, E.D. & MOIOLA, R.J. (1975) Geometry and growth of the White Sands dune field, New Mexico. *J. Res. U.S. geol. Surv.* **3**, 59–66.
- McKEE, E.D. & WEIR, G.W. (1953) Terminology for stratification and cross-stratification in sedimentary rocks. *Bull. geol. Soc. Am.* **64**, 381–390.
- MIDDLETON, G.V. & SOUTHARD, J.B. (1978) *Mechanics of Sediment Movement*. Soc. econ. Paleont. Miner. Short Course 3, Dallas, Texas.
- NIO, S.-D. (1976) Marine transgressions as a factor in the formation of marine sand wave complexes. *Geol. Mijnb.* **55**, 18–40.
- PETERSON, FRED & PIPIRINGOS, G.N. (1979) Stratigraphic relations of the Navajo Sandstone to Middle Jurassic formations, southern Utah and northern Arizona. *Prof. Pap. U.S. geol. Surv.* **1035-B**, B1–B43.
- PHILLIPS, R.L. (1979) Bedforms and processes on estuarine tidal-current ridge, Willapa Bay, Washington. *Bull. Am. Ass. Petrol. Geol.*, **63**, 509.
- POOLE, F.G. (1962) Wind directions in Late Paleozoic to Middle Mesozoic time on the Colorado Plateau. *Prof. Pap. U.S. geol. Surv.* **450-D**, D147–D151.
- PRATT, C.J. & SMITH, K.V.H. (1972) Ripple and dune phases in a narrowly graded sand. *J. Hydraul. Div. Proc. Am. Soc. civ. Engrs* **98**, 859–873.
- RUBIN, D.M. & MCCULLOCH, D.S. (1980) Single and superimposed bedforms: a synthesis of San Francisco Bay and flume observations. *Sedim. Geol.* **26**, 207–231.
- SALSMAN, G.G., TOLBERT, W.H. & VILLARS, R.G. (1965) Sand-ridge migration in St. Andrew Bay, Florida. *Mar. Geol.* **4**, 11–19.
- SAUNDERSON, H.C. & JOPLING, A.V. (1980) Paleo-hydraulics of a tabular, cross-stratified sand in the Brampton Esker, Ontario. *Sedim. Geol.* **25**, 169–188.
- SCHEIDEGGER, A.E. & POTTER, P.E. (1967) Bed thickness and grain size: cross-bedding. *Sedimentology*, **8**, 39–44.
- SIMONS, D.B., RICHARDSON, E.V. & ALBERTSON, M.L. (1961) Flume studies using medium sand (0.45 mm). *U.S. geol. Surv. Water-suppl. Pap.* **1498-A**, 76pp.
- SIMONS, D.B., RICHARDSON, E.V. & NORDIN, C.F., JR (1965) Bedload equation for ripples and dunes. *Prof. Pap. U.S. geol. Surv.* **462-H**, H1–H9.
- SMITH, J.D. (1977) Modeling of sediment transport on continental shelves. In: *Marine Modeling* (Ed. by E.D. Goldberg, et al.). *The Sea*, **6**, 539–577.
- SORBY, H.C. (1859) On the structures produced by the currents present during the deposition of stratified rocks. *Geologist*, **2**, 137–147.
- SOUTHARD, J.B. (1975) Bed configurations. In: *Depositional Environments as Interpreted from Primary Sedimentary Structures and Stratification Sequences*, pp. 5–43. Soc. econ. Paleont. Miner. Short Course 2, Dallas, Texas.
- STEIN, R.A. (1965) Laboratory studies of total load and apparent bed load. *J. geophys. Res.* **70**, 1831–1842.
- STOKES, W.L. (1968) Multiple parallel-truncation bedding planes: a feature of wind-deposited sandstone formations. *J. sedim. Petrol.* **38**, 510–515.
- STRIDE, A.H. (1965) Preservation of some marine current-bedding. *Nature*, **206**, 493–499.
- STRIDE, A.H. (1970) Shape and size trends for sand waves in a depositional zone of the North Sea. *Geol. Mag.* **107**, 469–477.
- SYKES, R.M. & BRANDY, R.P. (1976) Fan-delta sedimentation: an example from the Late Jurassic Early Cretaceous of Milne Land, central east Greenland. *Geol. Mijnb.* **55**, 195–203.
- WEBER, K.J. (1979) Computation of initial well productivities in eolian sandstone on basis of geologic model, Leman gas field, United Kingdom. *Bull. Am. Ass. Petrol. Geol.* **63**, 549.
- WILSON, I.G. (1972) Aeolian bedforms—their development and origins. *Sedimentology*, **19**, 173–210.

YALIN, M.S. (1964) Geometrical properties of sand waves. *J. Hydraul. Div. Proc. Am. Soc. civ. Engrs* **90**, 105-119.

ZNAMENSKAYA, N.S. (1963) Experimental study of the dune movement of sediment. *Sov. Hydrol., Select. Pap.* **3**, 253-275.

APPENDIX

In the development of equation (12) from equation (11), it is assumed that either H or L decreases linearly with distance downstream and that $(HL)_d \ll (HL)_u$. Assuming first that L decreases linearly, equation (11) reduces to

$$T \simeq \frac{H_u L_u}{2D} \quad (11a)$$

where L_u is related to L by the equation $L = \frac{1}{2}(L_u + L_d)$. Equation (12) is exactly equivalent to equation (11a) when $L_d = 0$, in which case H is constant, and when $L_d = L_u = \bar{L}$, in which case $H_u = 2\bar{H}$. To test whether equation (12) is a sufficiently close approximation of equation (11a) for values of L_d/L_u between zero and unity, an equation must be developed that relates \bar{H} , H_u , L_u , and L_d and that involves no approximation beyond those used in developing equation (11a).

By definition,

$$\bar{H} = \frac{1}{D} \int_{x_u}^{x_d} H dx.$$

Given the assumptions used in the development of equations (11) and (11a), and with no loss of generality

letting $x_u = 0$ and $x_d = D$, it can be shown that

$$H = \frac{ax+b}{cx+d}$$

where $a = (HL)_u$, $b = (HL)_u D$, $c = L_d - L_u$, and $d = L_u D$. Tables of integrals show that

$$\int \frac{ax+b}{cx+d} dx = \frac{ax+b}{c} + \frac{-ad+bc}{c^2} \ln(cx+d) + C$$

where C is the constant of integration.

Evaluating the integral between 0 and D and putting a , b , c , and d back into terms of H_u , L_u , L_d , and D ,

$$\bar{H} = \frac{H_u L_u}{(L_d - L_u)^2} \left[L_u - L_d + L_d \ln \left(\frac{L_d}{L_u} \right) \right].$$

Rearranging and substituting into equation (11a), it is found that

$$T \simeq \frac{\bar{H}(L_d - L_u)^2}{2D} \left[L_u - L_d + L_d \ln \left(\frac{L_d}{L_u} \right) \right]^{-1}. \quad (11b)$$

Equation (12) approximates equation (11b) to within 10% for any value of L_d/L_u from zero to unity. A similar result is obtained if H instead of L decreases linearly with distance downstream.

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